Collimation of the Newtonian telescope.

The Internet is full of guides on how to collimate Newtonian telescopes. Now I decided to write one more, because I got completely confused during my attempts to figure out why it is certain sequence of actions, and not the other. Is it necessary, as all techniques advice, to set the secondary mirror so that its center coincides with the center of the eyepiece tube? What is the displacement of the secondary mirror at high-aperture Newtonians? How to choose a diagonal mirror in order to get some pre-specified non-vignetted field of view, and what does "non-vignetted" even mean in general?

I think we should start with the optical design. All telescopes of the Newton's system have two mirrors, simply called "primary" and "secondary". The primary mirror for not very fast telescopes is made spherical, but for the apertures starting from $f/6$ and larger it will be much better to have a parabolic mirror. Newton's simplest circuit looks like this:

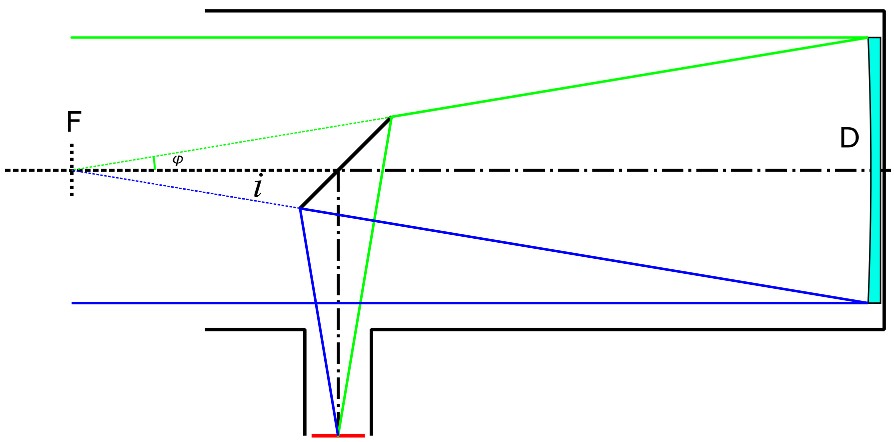

Figure 1 Optical scheme of a Newtonian telescope.

It can be seen that the main purpose of the main mirror is to collect light, and the secondary one brings the collected beam out of the tube so that it can be focused either on a photosensitive matrix or examined through an eyepiece. The secondary or so-called diagonal, mirror stands at an angle of 45° relative to the original optical axis, and its size is selected so that the primary mirror is completely visible in it, otherwise the light beams coming from its edges simply won't reach the focal point. It is this obstruction of light beams that is called vignetting.

It is obvious that the telescope will produce the highest quality image only if all the positions and angles of the primary and secondary mirrors are exactly as they were conceived by the manufacturer. Particular attention should be paid to the angles. And for astrophotographers it will also be extremely important that the optical axis after the reflection from the secondary mirror is strictly perpendicular to the sensor, since all possible skewing will lead to the fact that part of the field of view will be out of focus.

An attentive reader will probably notice that in Figure 1 the optical axis coming from the primary mirror divides the secondary one into non-equal parts. And this is not an illusion. The faster the telescope at our disposal, the stronger this effect will be manifested.

In order to estimate how large this offset should be, let's consider the area of interest in more detail, but first note that the light rays that initially fall on the main mirror parallel to the optical axis form a cone after the reflection and the point of convergence corresponds to the focal point of the main mirror. We will use this circumstance, since the problem of the shape, dimensions and position of the secondary mirror is now reduced to finding such a conic section that forms a plane passing at an angle of 45 degrees to the optical axis. The opening angle of our cone is determined by the focal ration of the main mirror i.e. the ratio of its diameter to the focal length; however, for further calculations it is more convenient to know half of this angle, which we will denote as φ:

$$\varphi = \operatorname{atan}\frac{D}{2F} \quad 1$$

Scheme which we will use for our calculation is presented below:

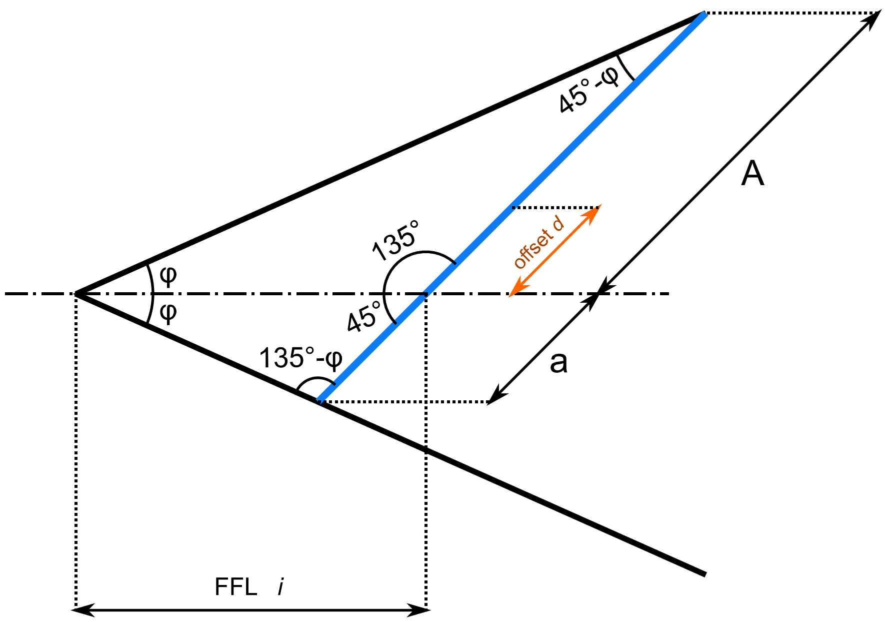

Figure 2 The geometry that defines the dimensions of the secondary mirror.

Figure 2, also shows all the known angles, as well as such a parameter as a focus-to-diagonal separation which I gonna call here “Flange focal length” or FFL. Its minimal value equal to at least half the diameter of the main mirror: since we want to move the focal plane outside the area from which we are trying to focus the light. It must also, include distance from the edge of the main mirror to the edge of the tube and the desired margin for the focuser movement. We will denote this total value in the current calculations as i.

The secondary mirror in Figure 2 is marked in blue. As you can easily see, its length is equal to $A + a$. That is what we want to find. As most people interested in astronomy know, when a cone is cut by a plane, we get either a circle, or an ellipse, or a parabola or a hyperbola. One can determine the type of the resulting curve by its eccentricity, determined by the formula:

$$\mathrm{e} = \frac{\cos\psi}{\cos\varphi} \quad 2$$

Where ψ is the angle of inclination of the secant plane to the axis of the cone, in our case 45 degrees. φ is the angle between the generatrix of the cone and its axis, i.e. the aperture of the cone is determined by the focal ratio of the primary mirror. However, in our case, to calculate the size of the secondary mirror along its long side, we do not yet need its eccentricity: firstly, based on the constructions, we already see that the section is closed, which means it can only be an ellipse, because our cutting plane is not perpendicular to the axis of the cone. And secondly, in order to find the major axis of our ellipse, we just need to remember the law of sines:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = k \quad 3$$

The ratio of the sines of the angles to the opposite sides for any triangle is a certain constant. So, for the upper triangle we can form a relation:

$$\frac{\sin\varphi}{A} = \frac{\sin(45° - \varphi)}{i} \quad 4$$

And for the lower one, correspondingly:

$$\frac{\sin\varphi}{a} = \frac{\sin(135° - \varphi)}{i} \quad 5$$

Thus, since we chose the FFL ourselves — we know its value (or it was given to us by the manufacturer of our telescope), and we can calculate the total length of the diagonal mirror by taking the sum $A + a$:

$$A + a = i \sin\varphi \left(\frac{1}{\sin(45° - \varphi)} + \frac{1}{\sin(135° - \varphi)}\right) \qquad 6$$

Or by using sine periodicity:

$$A + a = i \sin\varphi \left(\frac{1}{\sin(45° - \varphi)} + \frac{1}{\sin(45° + \varphi)}\right) \qquad 7$$

Using the formulas for the sum / difference of the sine angles, and also taking into account the fact that $\sin 45° = \sqrt{2}/2$ we can simplify the expression to a very simple formula:

$$A + a = i\sqrt{2} \tan 2\varphi \qquad 8$$

Or, using the formula for the double angle for the tangent and the fact that the tangent and arctangent are mutually inverse functions:

$$A + a = i\sqrt{2} \frac{2 \sin\varphi \cos\varphi}{\cos^2\varphi - \sin^2\varphi} = i\sqrt{2} \frac{2 \tan\varphi}{1 - \tan^2\varphi} = \frac{4\sqrt{2}DFi}{4F^2 - D^2} \qquad 9$$

It can be seen that the major axis of our diagonal mirror changes in proportion to the FFL. And the offset of the center of the mirror relative to the optical axis will also change proportionally, so it would be much more convenient to express the offset not in absolute, but in relative units. To do this, one needs to form the ratio:

$$r = \frac{d}{A + a} = \frac{A - a}{2(A + a)} \qquad 10$$

Value of $A + a$ we've already obtained in 9. Now let's find $A - a$:

$$A - a = i \sin\varphi \left(\frac{1}{\sin(45° - \varphi)} - \frac{1}{\sin(45° + \varphi)}\right) \qquad 11$$

Or, simplifying:

$$A - a = i \left[\frac{\sqrt{2} \sin^2\varphi}{(\cos 2\varphi)}\right] \qquad 12$$

Substituting to the 10 values from 8 and 12 we get:

$$r = \frac{A - a}{2(A + a)} = \frac{i\sqrt{2} \sin^2\varphi}{\cos 2\varphi} \cdot \frac{\cos 2\varphi}{2i\sqrt{2} \sin\varphi \cos\varphi} = \frac{\tan\varphi}{2} \qquad 13$$

Finally, taking into account 1, we get this:

$$r = \frac{D}{4F} \qquad 14$$

So, the secondary mirror offset in fractions of its major axis is equal to the focal ratio of the primary mirror, divided by four. Regardless of the size of the FFL.

Or, if we still want to get the formula for the absolute value of d:

$$d = r(A + a) = \frac{\sqrt{2}D^2 i}{4F^2 - D^2} \qquad 15$$

In the eyepiece tube, when looking from the focus point, we will see that the diagonal mirror will be set strictly concentric to the focuser tube, and its slope will need to be adjusted so that it completely reflects the main mirror. However, there is a catch: let's say that our focuser is a little crooked and its axis isn't strictly perpendicular to the telescope tube, but at a slight angle. At the same time, the focuser tube itself does not bend when moving, and the camera sensor still remains perpendicular to it. Does this mean that the focuser should be thrown away / changed / fixed? In principle, no, we still have some hope. If this deviation does not exceed one or two degrees, this is of course unpleasant, but quite fixable: you can simply change the slope of the diagonal mirror, making it not 45 degrees, but a little more or less, according to the situation, as in Figure 3.

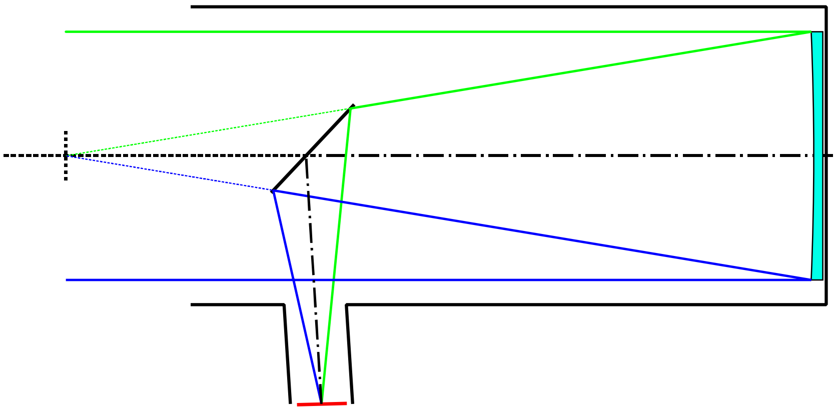

Figure 3 An illustration of a possible correction for an error in the manufacture of a focuser.

After such adjustment, of course, our Newton turns into a kind of hybrid either with the Herschel system, or a hybrid with brachite. Naturally, for illustration, a frankly bad focuser was taken, with a deviation of three and a half degrees, which leads to a deviation of the secondary mirror from the calculated position by 1.75 degrees, respectively. At the same time, it can be seen that the requirement for the concentricity of the secondary and the ocular tube when viewed from the focus point is preserved. The secondary mirror simply shifted from the calculated point a little further from the primary mirror, reducing the FFL. From this case we can conclude: it is not the angle of the secondary relative to the main mirror itself is important, but the correctness of its position relative to the focuser. Although it is obvious that a proper focuser at least does not complicate the alignment process.

Here, however, a very important point must be made! This method of correcting the focuser error only works for one type of skew. If the focuser has a tilt only towards or away from the main mirror, this method of correction with the adjustment of the angle of the secondary mirror will work. If, however, we oriented the tube with its end facing us, and the extension of the focuser axis runs not along the diameter of the tube, but along the chord, then this requires an unequivocal correction, since no tilt of the secondary mirror will help correct the misalignment of the focal plane in this case. That is why many alignment manuals recommend starting alignment not even with mirrors, but with setting the focuser in such a way that its axis crosses the axis of the telescope tube. It's easier to understand this with a diagram:

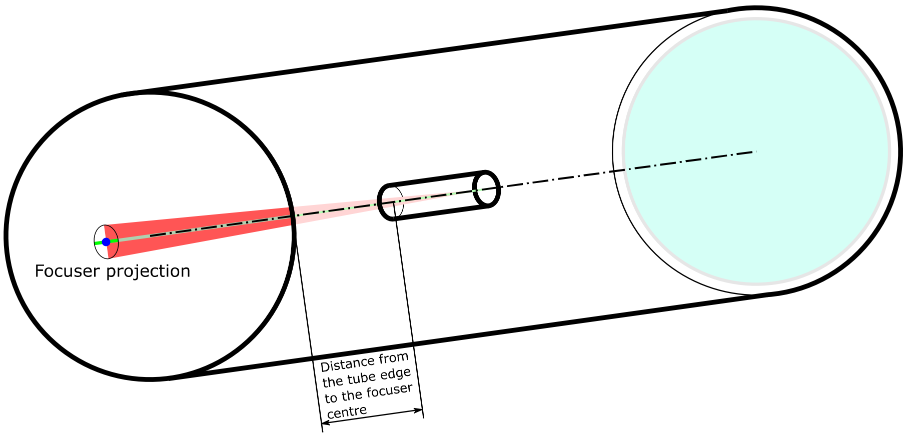

Figure 4 An illustration of which focuser tilts can be corrected with a secondary mirror.

Green in Figure 4 indicates correctable tilts of the focuser. As you can see, they must lie in the plane in which the optical axis of the main mirror is located. Red, respectively, indicates invalid positions.

This is precisely why many astronomers first remove the secondary mirror and the focuser too, and mark the geometric center of the hole remaining from the focuser on the opposite side of the tube. Sometimes, they even by drill a tiny hole there! This point is marked in blue in Figure 4.

And this entire ordeal is just to ensure that the axis of the focuser tube crosses the diameter of the main tube. Nothing can beat the perfection! Since we're trying to minimize focuser tilts anyway, why stop halfway? This goal is achieved by twisting the focuser adjustment screws, if any, or by placing washers or any other spacers between the focuser base and the main tube. Although, as I have shown, the tilts of the focuser, parallel to the optical axis of the main mirror, can be corrected.

Before proceeding, to the collimation itself, I would like to note that all those calculations that were carried out to calculate the shape, size and position of the secondary mirror were made under the assumption of a zero non-vignetted field. And this is obvious. Figure 2 clearly demonstrates that, for example, for the upper beam, any deviation above the diagonal mirror leads to the fact that it is no longer reflected and doesn’t participate in the construction of the image. Some planetary observers design their Newtonians specifically for a zero nonvignetted field, since this allows the size of the secondary to be reduced as much as possible, and, consequently, to reduce the value of the central obstruction, which leads to a slight weakening of diffraction effects. Usually telescopes have a secondary mirror of such dimensions that the unvignetted field is within 5-15 mm range, which is quite enough for visual and photographic purposes and still doesn’t cause severe image degradation. To recalculate our formulas for this case, as it turns out, very little needs to be changed.

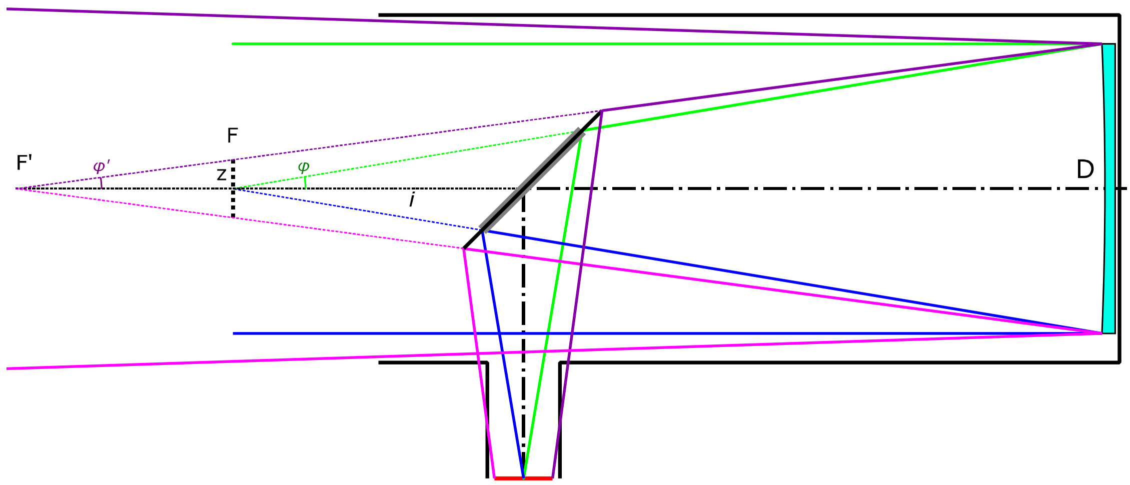

Figure 5 Calculation of a diagonal mirror for a non-zero non-vignetted field.

Figure 5 shows that for the non-vignetted field z we have chosen, some of the rays simply did not fit into the focuser: it was chosen so large that either we had to curb our appetites or change the equipment. This is understandable: a 40 mm non-vignetted field (not with such a FFL) could not fit in a two-inch focuser. In principle, for the calculation, you only need to set the size of the desired field z, and find where our rays will now converge, forming a fictitious focus point, thus having a new angle φ'. To find it, we use the similarity of the obtained triangles. And then the procedure is the same. Calculate the angle for the new fictitious focal ratio:

$$\tan\varphi' = \frac{z}{2(F'-F)} = \frac{D}{2F'} \quad 16$$

Finding F' from the last two equalities:

$$F'z = F'D - FD$$
$$F'D - F'z = FD \quad 17$$
$$F' = \frac{FD}{D-z}$$

The new angle will be:

$$\tan\varphi' = \frac{D-z}{2F} \quad 18$$

Correspondingly the new secondary mirror size is:

$$A + a = i'\sqrt{2}\frac{2\tan\varphi}{1-\tan\varphi^2} = i'\sqrt{2}\frac{2\frac{D-z}{2F}}{1-\left(\frac{D-z}{2F}\right)^2} = i'\sqrt{2}\frac{4F(D-z)}{4F^2-(D-z)^2} \qquad 19$$

Where i' — is a fictitious FFL: the distance from the "new focal point" to the secondary mirror. It is evident that the real FFL is shorter than a fictitious FFL by $F' - F$, which has to be accounted for in our formula:

$$i' = i + F' - F = i + \frac{Fz}{D-z}$$

$$A + a = \left(i + \frac{Fz}{D-z}\right)\sqrt{2}\frac{4F(D-z)}{4F^2-(D-z)^2} \qquad 20$$

$$A + a = \frac{4\sqrt{2}F[iD + z(F-i)]}{4F^2-(D-z)^2}$$

Our new r will still be calculated from the angle, but from the φ', instead of φ, therefore we gonna have a very simple formula for it:

$$r = \frac{\tan\varphi'}{2} = \frac{D-z}{4F} \qquad 21$$

Or, if we still want to find the offset directly in millimeters:

$$d = r(A+a) = \frac{\sqrt{2}[iD + z(F-i)](D-z)}{4F^2-(D-z)^2} \qquad 22$$

The most compact expression for a non-vignetted field, if we take for the secondary mirror as a measure of its size its major axis, I managed to get it in this form:

$$z = D - \frac{2F\left[\sqrt{2}(F-i) - \sqrt{B\left(B-\sqrt{2}D\right) + 2[F-i]^2}\right]}{B} \quad \text{where} \quad B = A + a \qquad 23$$

But what happens if we try to write it based not on the major axis of the ellipse, but on the minor one? It turns out that if we assume that the minor axis of our elliptical mirror is a square root of two times smaller than the major axis $b = B/\sqrt{2}$, then expression 23 can be further reduced:

$$z = D - \frac{2F\left[\sqrt{2}(F-i) - \sqrt{b\sqrt{2}\left(b\sqrt{2}-\sqrt{2}D\right) + 2[F-i]^2}\right]}{b\sqrt{2}} \qquad 24$$

$$z = D - \frac{2F\left[(F-i) - \sqrt{[F-i]^2 - b(D-b)}\right]}{b}$$

An attentive reader, however, may ask a legitimate question: "Why did we decide that the aspect ratio of our elliptical mirror is equal to the root of two? Yes, it stands at an angle of 45 degrees, but after all, in formula 2, which just gives us eccentricity, we can get other values.

This remark is legitimate, but let's see what real values of eccentricities we can get for our secondary mirrors. As mentioned earlier, ψ is the angle of inclination of the mirror in our case is always 45 degrees, and φ is the angle determined by the focal ratio of the main mirror.

On the left is a graph showing the values of the eccentricity of the secondary mirror depending on the focal ratio of the telescope, and on the right is a demonstration picture of how little the calculated shape of the mirror for aperture $f/2$ differs from those mirrors that have a standard aspect ratio of $1{:}\sqrt{2}$. It can be seen that even for a ludicrously high focal ratios such as $f/2$, the deviations do not exceed a few percent. In practice, the secondary mirrors of most telescopes are chosen of such sizes that an unvignetted field is present. From the fact that the mirror is half a millimeter shorter along one of the semi-axes, our vignetting along one axis will simply increase more than along the other by those very fractions of a percent, which will be absolutely impossible to notice. This means that our formula 24 can be considered correct with a high degree of accuracy.

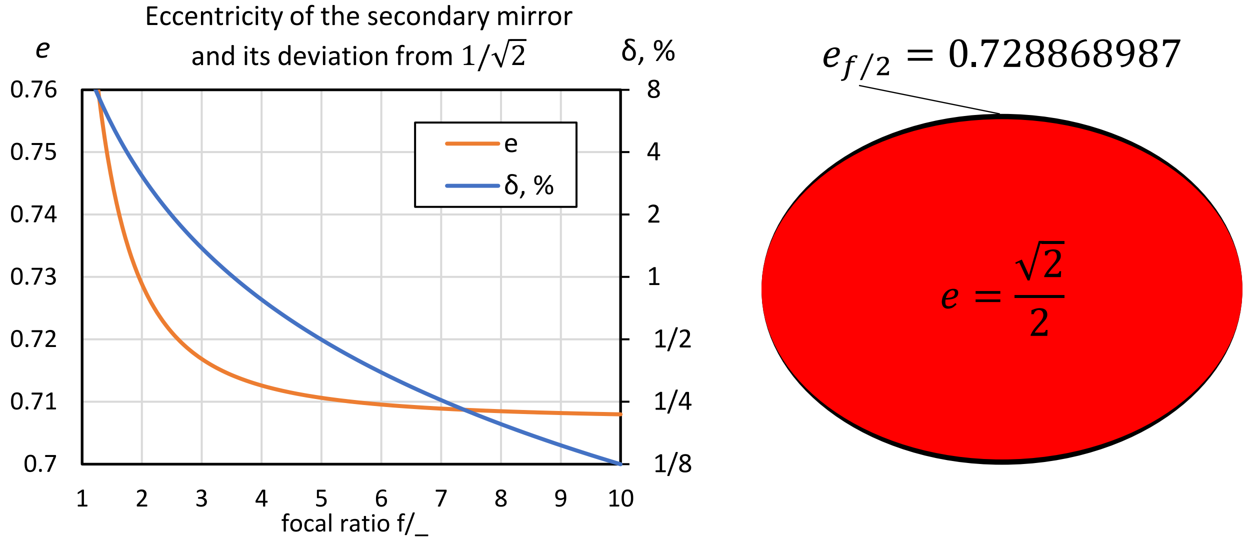

Figure 6 Estimation of the eccentricity of a secondary mirror.

Also, using the simplified formula 24, one can derive a formula for the relative displacement of the diagonal, containing only the dimensions of the mirrors and the distances between them:

$$r = \frac{(F - i) - \sqrt{[F - i]^2 - b(D - b)}}{2b} \qquad 25$$

However, let's get back to alignment. Let's assume that we either completed the stage of setting the focuser, or relied on the honesty of the manufacturer. The first thing we must do next is to set up the secondary mirror so that, when viewed from the focal point, it is concentric with the eyepiece tube and at the same time fully reflects the primary mirror. And the requirement that we should look exactly from the focus point when adjusting, or at least from the optical axis, is quite strict. Since at this stage **any** displacement from the optical axis will inevitably lead to the non-concentricity of the secondary relative to the focuser, and as a result, the focal plane will become non-orthogonal to the focuser axis. Which will lead to the already described effect: it will be impossible to get sharpness throughout the frame. When the focuser moves, the focus on the photo will roll from one place to another. Therefore, it is strongly recommended to use a Cheshire eyepiece for alignment. And for further improvement of the accuracy, I would recommend putting a tiny mark on the secondary mirror itself, which would show not its geometric center, but the point through which the optical axis passes, and which we calculated taking into account the displacement using the formula 14, or, for a real case with a non-vignetted field — 21. Then, having either a Cheshire eyepiece with a crosshair or a laser collimator, we could easily set the secondary mirror to the desired position using this mark, and then we would only have to choose such its tilt to see the full reflection of the main mirror.

Up to this point, I somewhat omitted the method of attaching the secondary right in the center of the tube. But any owner of a newton knows that it does not hang there by itself, but holds it in place with a special mount, also known as a “spider” (Further I’ll omit the quotation marks). In fact, these are four (sometimes three or even one) thin steel plates. By rotating the screws with which they are attached to the tube, it is possible to move the diagonal mirror horizontally within certain limits. And vertical displacement is possible due to the secondary mirror holder, to which the diagonal frequently is simply glued. The inclination is controlled through three adjustment screws passing through the bushing located in the spider itself and resting against the holder.

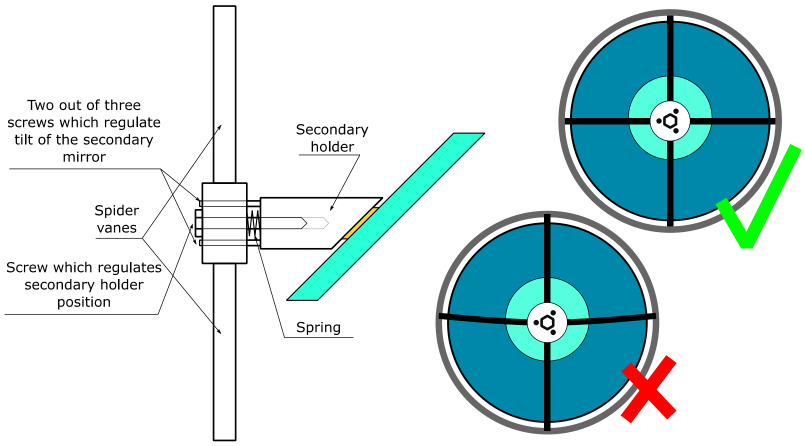

Figure 7 Scheme of mounting the secondary mirror with the correct and incorrect spider alignment. On the left is a frontal view of the tube, showing two options for aligning the secondary.

When looking at the figure, it is immediately clear that although spider vanes allow you to move the secondary horizontally, it is strongly advised to do this subtly, since curved vanes will lead to doubling and amplification of the rays from the stars due to diffraction, which in Newtons is absolutely always present. And that it is better to make them the same length, and glue the secondary to the holder correctly in advance, taking into account the displacement formula, to which the entire text was devoted earlier.

Well, at the end of our adjustment, if all the previous conditions have been met, we will need to move the mark on the main mirror into the reflection of the black eye of the Cheshire eyepiece, using the adjustment screws of the main mirror, of course. So, as a final result of the adjustment, we will have to get the following picture:

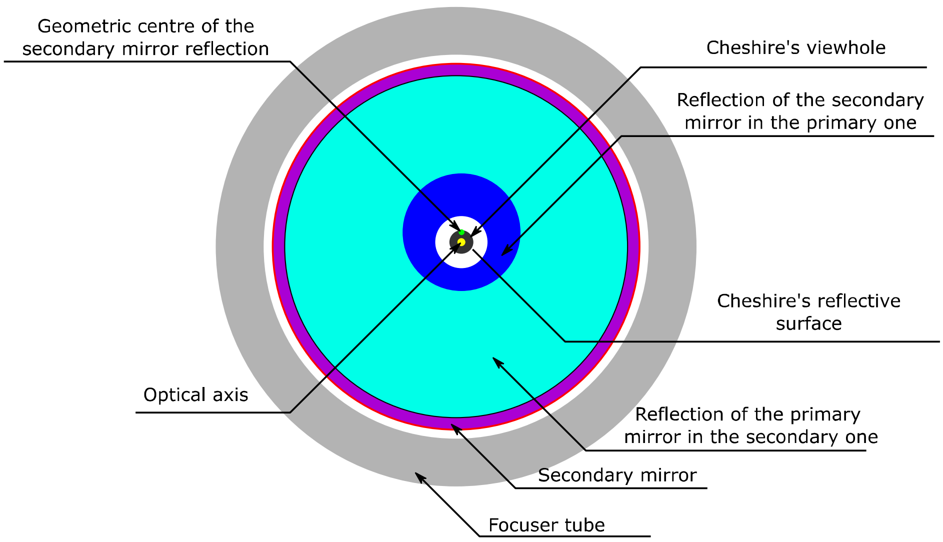

Figure 8 Schematic view when looking through the Cheshire eyepiece with the correct alignment. Spider legs, legs which hold the main mirror, and central marks of both mirrors are not shown.

It can be seen that the reflection of the secondary mirror in the primary one is shifted, but this is **not** an alignment error. That is how it should be! And the greater the focal ratio of your Newton, the stronger this shift will be pronounced. It is precisely that offset, we calculated using formula 14. And it can be larger than in the picture only in two cases:

1. You are the owner of a telescope with the focal ratio greater than $f/3$, because the figure above shows the ideal collimation exactly for this case.
2. The secondary mirror was glued by the manufacturer incorrectly, and he was not guided by formulas, just eyeballing instead. Then I would strongly recommend cutting it out and re-gluing it yourself. Maybe for visual observations this will not interfere much, but on astrophotography images it will result in the fact that the

brightest part of the frame will be shifted to the edge of the matrix. Flat field shots can certainly improve the situation, but the signal-to-noise ratio for the darkest areas will be worse than it could be. Plus, if the secondary is initially glued with a significant deviation from the calculated offset, then when we set it concentrically to the focuser tube, and then we play with its inclination in order to fit the reflection of the main mirror in it, we will set not its calculated angle, but a different one. Which again will lead to an inclined focal plane. How critical this is for alignment will be discussed later. I consider it important to repeat that at fast newtons we will never get such a picture:

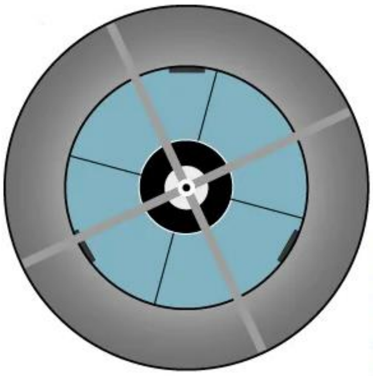

Figure 9 Picture demonstrated in many manuals as an example of correct alignment, unattainable at fast Newtons under any circumstances. The black silhouette of the secondary in this picture will be displaced, as mentioned earlier.

For “dark” (aperture less than $f/6$) newtons, the offset of the secondary may indeed be insignificant and imperceptible to the eye, but when this picture is shown in beginner manuals, as for me, this definitely leads only to delusions.

Now that the main steps have been taken apart, we can try to figure out how precisely in general we need to carry out all these adjustments? Maybe one/two millimeters/degrees won’t lead to any catastrophe? This is what we will try to evaluate.

The first adjustment step is, as we found out, setting the secondary mirror concentric with the focuser tube. But what if it was originally glued to the holder incorrectly? Without the offset we calculated, or even with an offset in the opposite direction? Let's first try to evaluate this visually on the diagram, and then put it into numbers. I will perform all the calculations according to the formulas for the case of a zero non-vignetted field, i.e. as if we have a diagonal mirror of the minimum size. This greatly simplifies the calculations, but it does not affect the result qualitatively, since, for example, the displacement of a diagonal mirror gives values of r that differ by only by:

$$\delta r = \frac{r - r'}{r} = \frac{z}{D} \sim 10 \div 15\% \qquad (26)$$

At the same time, it must be remembered that a 10% correction is not for the angles themselves, but for their **tolerances**.

Also, whenever possible, I will now use the approximation that the tangent of a small angle is approximately equal to the angle itself **in radians**. Starting from apertures smaller than $f/3$, such an approximation will also introduce an error less than a percent into the calculations:

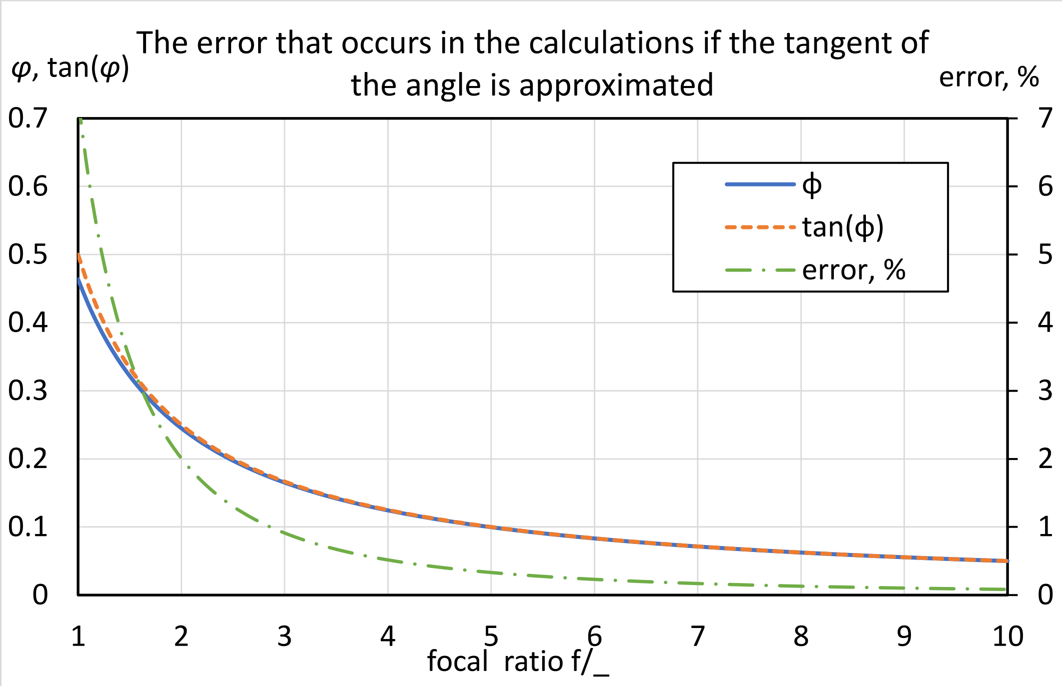

Figure 10 Evaluation of the error due to replacing the value of the tangent by the angle in radians.

Suppose that our manufacturer is frankly crooked and stuck a diagonal not only without offset, but also with an offset in the wrong direction. Where does it lead? To the fact that it will effectively move the diagonal further from the main mirror and closer to the focuser. If, at the same time, we shift it by moving the holder back so that it stands concentric with the focuser tube, and keep its angle of inclination at 45 degrees, and, let me remind you, we don't know it in advance, we will see that the reflection of the main mirror in the secondary is cut… We gonna try to correct the angle so that the reflection of the primary fits in and becomes concentric with the secondary, as all the manuals tell us, and precisely after that the focal plane will get inclined! That is why I spent so much time deriving a formula that calculates the correct position of the diagonal, since this is a very important parameter.

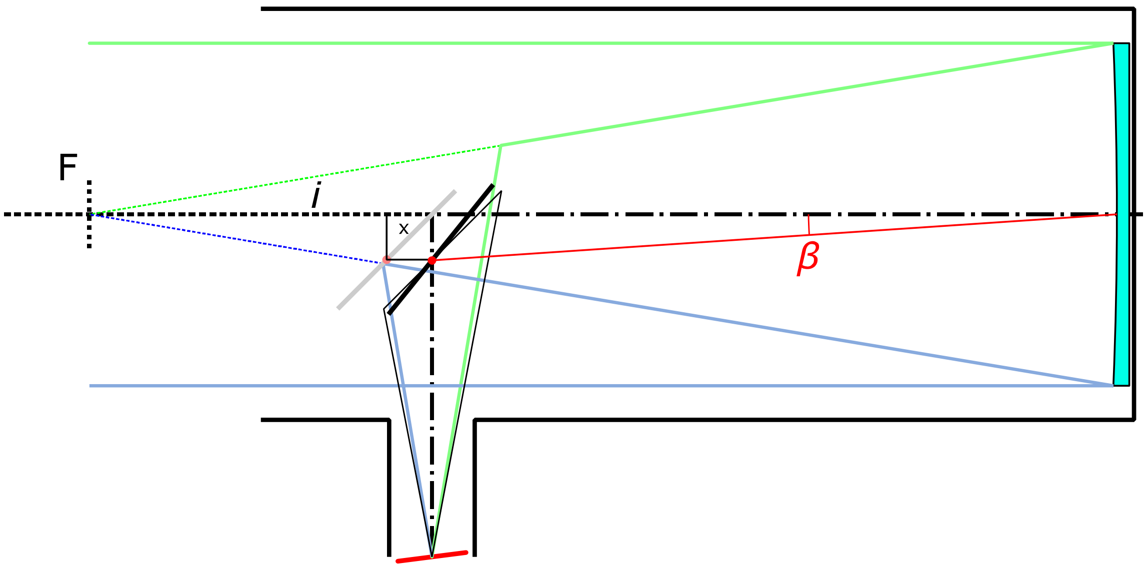

Figure 11 Newton collimation with the incorrectly glued secondary. Gray line shows where it should be (with the wrong gluing). Thin lines represent the position after the concentric alignment with the focuser tube. The position after turning in such a way that the main mirror is completely reflected — a thick line. The red label shows the correct offset d.

All labels in Figure 11 correspond to those introduced earlier, except for two: the first new variable is x, the value of the deviation of the position of the secondary mirror relative to the calculated displacement d. If it were glued correctly, it would be equal to zero. And the second is the angle β formed by a triangle, in which one of the legs is equal to the distance from the point of correct gluing of the secondary mirror to the optical axis of the main mirror, and the second leg is the distance from the projection of the focuser center onto the optical axis of the main mirror to the center of the main mirror. The corresponding formula represents these distances more clearly:

$$\tan\beta = \frac{\frac{x}{\sqrt{2}}}{F-i} = \frac{x}{\sqrt{2}(F-i)} \approx \beta \qquad 27$$

It is not difficult to figure out, and as you can see from the figure, the secondary mirror will need to be rotated by half this angle in order to maintain the illusion of correct alignment of the secondary. And it is precisely at this same angle β that the focal plane will incline with respect to its initial position.

How critical is this? What angle accuracy do we need to get?

The simplest qualitative estimate can be obtained from the following considerations: rays arrive at each pixel in the form of a cone, with the opening angle already familiar to us, corresponding to the focal ratio of our telescope. But now we need not half but the whole angle:

$$\operatorname{atan} 2\varphi = \frac{D}{F} \Rightarrow 2\varphi \approx \frac{D}{F} \qquad 28$$

Also, it does not hurt us to know the angular resolution of our telescope, which is determined by the diameter of the mirror. According to the Rayleigh criterion, we have:

$$\alpha = \frac{140"}{D(mm)} = \frac{6.8\cdot 10^{-4}\, rad}{D(mm)} \qquad 29$$

Now suppose that we have focused the center of the image, but at the same time we have a matrix skew relative to the focal plane, that is, the following picture is observed:

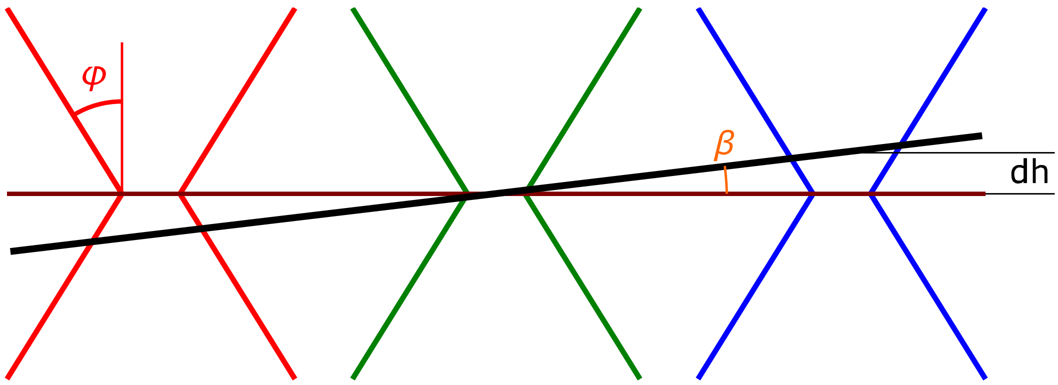

Figure 12 Rays of light arriving at a matrix tilted at an angle β relative to focal plane.

The rays in Figure 12 do not converge to a single point due to diffraction. We determined the diffraction limit using formula 29. Knowing the focal length of the telescope, we can estimate the linear size of these diffraction spots (although it would be more correct to say that this is not the size of the spots, but the size of the Airy disk — the central maximum of the diffraction pattern):

$$s_{diff} \approx \alpha F \qquad 30$$

Deviation from the focal plane, both positive and negative, only leads to an increase in our spots. We also know that the minimum spot size is equal to s_{diff}, and it grows in proportion to the deviation from the focal plane dh, and the angle $2\varphi \approx D/F$ serves as our coefficient of proportionality. From this you can get the following qualitative calculations:

$$dh \approx l\beta/2$$
$$s(h) = s_{diff} + \frac{D}{F}|h| \qquad 31$$
$$s(\beta) = s_{diff} + \frac{D}{F}\left|\frac{l\beta}{2}\right|$$

Where $l/2$ half the size of our matrix (in millimeters) along the long side if we focused on its center. How can we derive the tolerance for the angle β from here?

For a start, let's try to evaluate the focusing tolerance with our qualitative formulas. Let's assume that the faintest stars left a trace of 2x2 pixels on the matrix. If on the edge the smallest stars form at least a 3x3 pixel circle, then most likely we will notice this. Let us take as the value of the allowable swelling of stars for $t = 0.5$:

$$\frac{D}{F}|h| \le ts_{diff} \quad \Longrightarrow \quad |h(\mu м)| \le 6.8 \cdot 10^{-1} t \left(\frac{F}{D}\right)^2 \qquad 32$$

For the focal ratio of $f/6$, the deviation is $|h| \le 12\mu m$, and for $f/4$ it is already $5.5\mu m$. However, our formulae do not take into account the effects of diffraction, since the light reflected from the main mirror forms not just a converging beam, but also a diffraction pattern.

Without going into mathematical details, in a nutshell, we can say that the diffraction pattern is calculated based on the Huygens principle, which states that each point of the front (or surface reached by the wave) is a secondary source of spherical waves. All these waves are summed up and give the total intensity of light at the desired point. And to calculate this sum, the Fresnel summation principle comes into play, when the surface under study is divided into many small segments, the boundaries between which are chosen in such a way that the ray path difference between adjacent segments is equal to half the wavelength.

There is a formula for the error of the wavefront formed by the main mirror when reflected not into focus (F), but near it ($F_d = F + \Delta$):

$$W_d = \left[\frac{1}{F_d} - \frac{1}{F}\right]\frac{D^2}{8} = \frac{D^2}{8}\left[\frac{F - F_d}{FF_d}\right] = -\frac{D^2}{8}\frac{\Delta}{FF_d} \qquad 33$$

For a small defocus, when $\Delta \ll F$ we can say that $F \approx F_d$, and if we remember that F is the distance to the focus point, and D is the aperture of the main mirror, then it turns out that due to defocus, the wavefront error is directly proportional to deviation from the focus point and the square of the focal ratio:

$$W_d = -\frac{D^2}{FF_d}\frac{\Delta}{8} \sim \frac{\Delta}{8}\left(\frac{D}{F}\right)^2 \qquad 34$$

Usually, the distance at which defocus is considered to be negligible can be taken such a value Δ where the wavefront error is less than $W_d < \lambda/4$. Substituting this value into 34 we get:

$$\frac{\Delta}{8}\left(\frac{D}{F}\right)^2 \lesssim \frac{\lambda}{4} \quad \Longrightarrow \quad \Delta \lesssim 2\lambda\left(\frac{F}{D}\right)^2 \qquad 35$$

Taking the wavelength of light as 500 nm, we find that the offset from the focus point in microns is permissible by an amount inversely proportional to the square of the focal ratio. That is, almost 3 times more than according to 32. One should take into account though that the value of Δ calculated in this way implies a two-sided interval: $\lambda/8$ in each direction. When the deviation is greater than that we will observe how the energy is redistributed in the diffraction pattern: the central maximum is weakened and the rings surrounding are getting more pronounced.

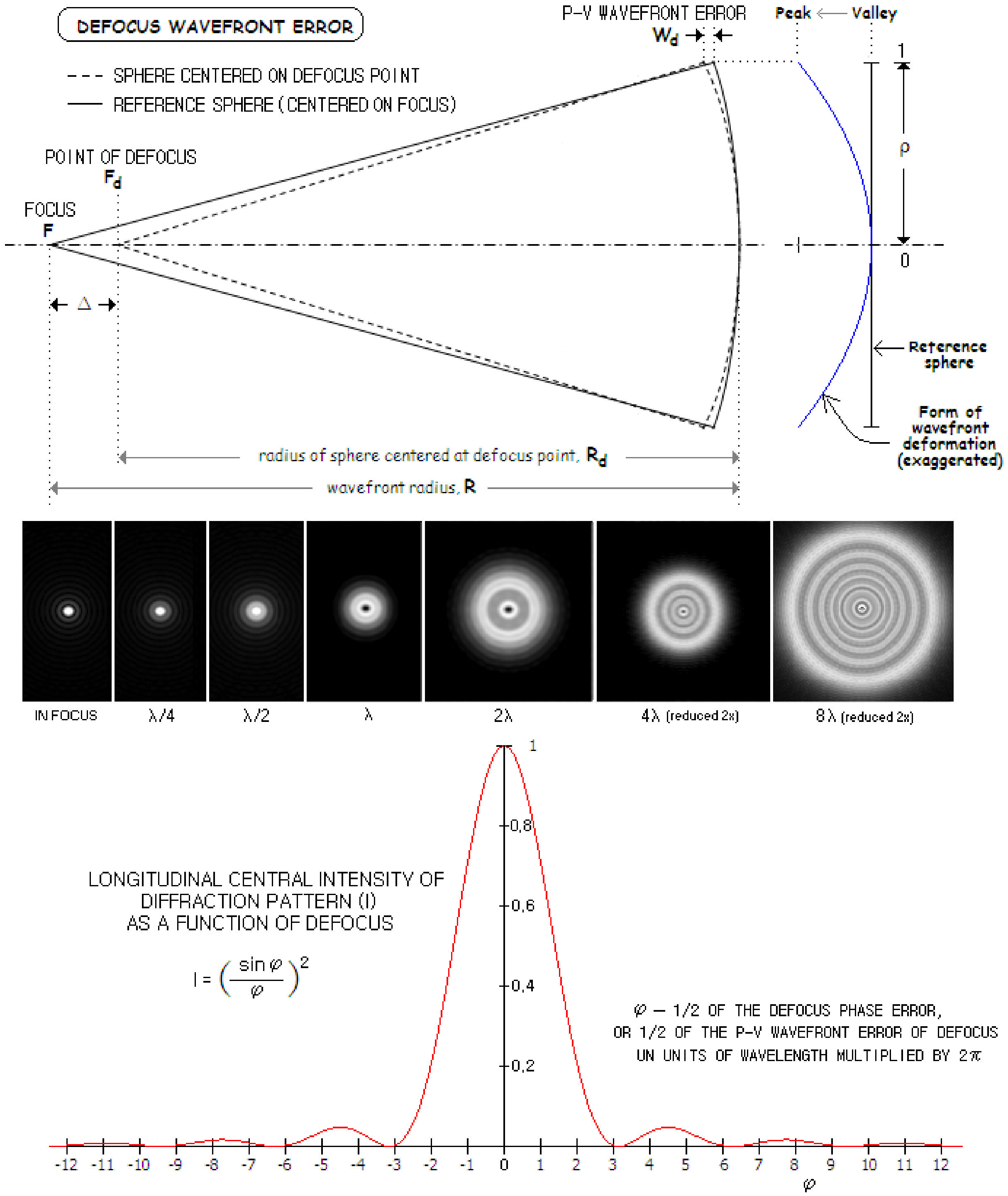

Figure 13 Illustration of the diffraction effects arising due to wrong focus.

There are programs that allow you to calculate these effects. For example, a free aberrator. The possibilities of this program, with a size of modest three megabytes, are quite high. It is able to simulate effects that can be observed through a telescope, such as a diffraction pattern, simulate blurring of an image due to atmospheric jitter or optical defects: spherical aberration of various orders, coma, astigmatism, and others. It is also possible to construct a cross-section of the diffraction pattern there when moving from the pre-focal to the out-of-focal position, including taking into account the central screening of the secondary mirror. Pictures resulting from such calculations can be seen below.

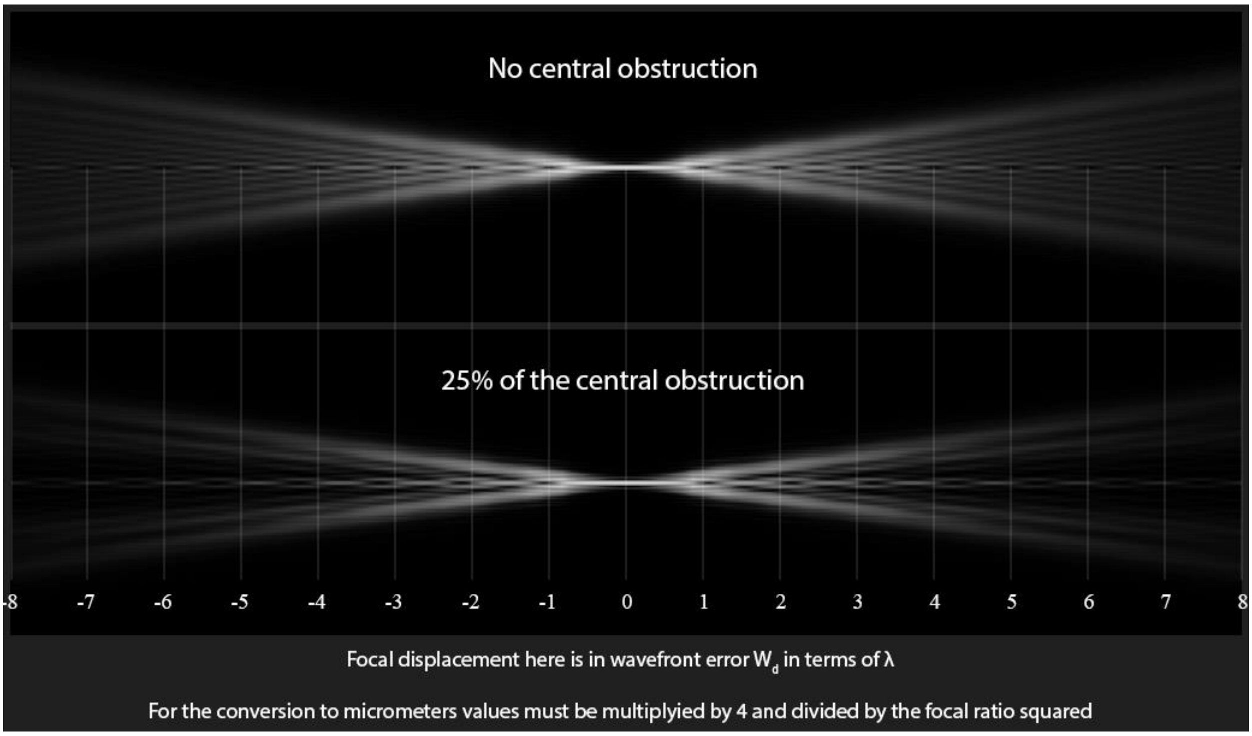

Figure 14 Cross section of the diffraction pattern when moving through the focal point.

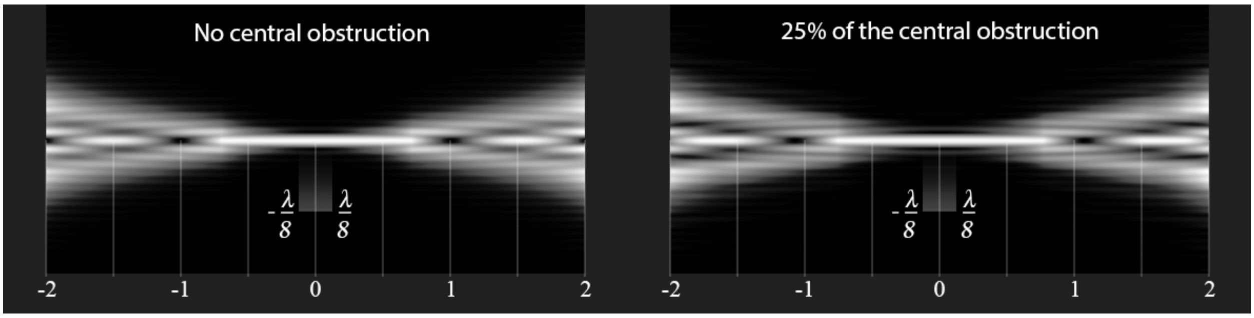

Figure 15 Detailed section near the focus. The desired focus range is marked. There the quality is limited only by diffraction.

In Figure 15, you can see that if there is a central obstruction in the system, then the second diffraction ring will be a little brighter. Plus, the usable area of the mirror that can collect light is also reduced. Therefore, chasing a large size of the non-vignetted field, so that the entire matrix is within its limits, even in the case of an astrograph, frequently is not worth it.

Now that we have determined the required defocus tolerance, calculating the maximum focal plane tilt angle is trivial:

$$dh \approx l\beta/2 \quad \Longrightarrow \quad |\beta| \leq \frac{2\Delta}{l} \qquad 36$$

So, the tolerance for the angle of the secondary will be equal to the value from the formula 36:

$$|\beta_{rad}| \lesssim \frac{2 \cdot 10^{-3}}{l(mm)} \left(\frac{F}{D}\right)^2 \quad |\beta_{degree}| \lesssim \frac{0.114}{l(mm)} \left(\frac{F}{D}\right)^2 \qquad 37$$

Now you can get some numbers from which you can start. For example, on my telescope with the focal ratio defined as $150/1000$ on a full-frame sensor ($36 \times 24\ mm$), I need to set the diagonal mirror and focuser with an accuracy of about nine arc-minutes so that the final skew of the focal plane does not introduce a noticeable blur at the edges of the field. At the $f/4$ aperture for a full frame, this accuracy should already be better than three arc-minutes. For APS-C format matrices with dimensions of 22.2×14.8, these values can be increased in proportion to the crop factor: 1.6 times. With such accuracy, we need to collimate our telescope.

From here, for example, it can be seen that if the secondary is glued crookedly, and according to formula 27, if the FFR is $i \approx F/4 \div F/5$, then the angle β should not exceed three and a half arc-minutes, or 1 milliradian:

$$x \lesssim \beta\sqrt{2}(F - i) \approx 1.1F\beta \qquad 38$$

For a typical focal length of 1 meter, an acceptable displacement of the secondary mirror is around one millimeter with respect to its calculated position, or preferably smaller. It is obvious that if the tolerance for star swelling is weakened (which I would not recommend), then all other estimates will increase proportionally, but the order of magnitude will remain: the accuracy of mirror inclination is about a few arc minutes, and the accuracy of gluing and positioning the secondary should be millimeter-like.

Gluing the secondary with an offset to the holder, maintaining millimeter accuracy, is not a daunting task. But putting it in the right position with the same accuracy will be more difficult. Also, we should not forget that the tilt of the focuser, discussed earlier (Figure 4), can also lead to an inclined focal plane. Therefore, the accuracy of its alignment should be comparable to the accuracy of the position of the secondary mirror, or even higher. So, if the focuser's landing pad has a distance between the holes for attaching to the main tube, say 80 millimeters, then a millimeter difference in height will lead to an angle of $\beta \approx 43'$! Therefore, the adjustment of the focuser must be taken seriously, eliminating all possible distortions as much as possible even before the procedure for selecting the position of the secondary mirror. This, if I may say so, is the zero stage of adjustment.

After the focuser is aligned, and the secondary mirror is either re-glued, or we made sure that the manufacturer did everything right, the stage of selecting its position in the tube begins. As we have already seen, fulfilling this task by eyeballing with the required precision will be absolutely impossible. As a first approximation, you can use the Cheshire eyepiece. It will limit the freedom of movement of the observer's eye and force us to look, albeit not strictly from the axis of symmetry of the focuser, but at least it won't allow us to deviate further than 2-3 mm from it… There are two ways to go: the first assumes that we are trying to observe concentricity and rely on our eye.

Here, it is **highly** desirable to look not from any distance, but from a focus point, because the offset from formula 25 was calculated for this position. An error in the position of the eye relative to the center of the focuser tube of 1-2 millimeters for an FFL of 250 mm will result in an angle error approximately equal to:

$$\beta = \frac{(1 \div 2)mm}{i(mm)} \approx 15' \div 25' \qquad 39$$

This result can be slightly improved if we go the second way. Here, as I suggested earlier, we put a small mark on the secondary mirror, marking its offset from the geometric center, and then, either using the standard crosshair of the Cheshire eyepiece, or making our own crosshair from improvised materials in it, we put the secondary so that the mark and crosshairs matched. At the same time, since we no longer use such a parameter as the concentricity of the secondary and the focuser, we can additionally increase the distance from the observation point. However, it is clear that in this way the angle β is unlikely to be reduced by more than one and a half to two times, since this will already involve extension tubes of almost 25 cm in length. And at the same time, we should not have sagging of the focuser under such a lever… Although even reducing the angle β to $10'$ will already allow you to get a pretty decent field even on a full-frame matrix with aperture up to $f/6.7$, and with cropped matrices up to $f/5$.

If we entrust the task of sighting not to the eye, but to the laser collimator, then the uncertainty in the position will be at least twice as small, which further reduces the error in the angle β. In case we made sure that the direction of the collimator beam is coaxial with the focuser tube that is. If the collimator manufacturing process wasn't perfect, then when the laser rotates in the focuser, the point will not remain in the same place, but will trace a circle on the main tube wall, or on our secondary mirror… This is of course sad, but you can outline a full circle with this laser, understand where its center is and drive the mark on the secondary into this, now virtual, position. Although of course this is a potential loss in accuracy.

Further, after the correct position of the secondary is established, we select its slope so that you can see the reflection of the primary mirror in the secondary in its entirety. Naturally — it must be concentric. You can rely on both the contour of the mirror itself, or on its clips that hold the main mirror in its frame. Moreover, if we look not from the focus point F, but from our virtual focus F', corresponding to the point where the non-vignetted field disappears, then it will be even easier to maintain the concentricity of the secondary mirror and the desired reflection than when observing from the focus point. Well, as a third way — you can drive the laser point from our collimator again into the center of the primary mirror. All three methods are equivalent here.

And as a final step, with the adjustment screws of the main mirror, you need to drive the reflection of the black eye of the Cheshire eyepiece into the central mark on the main mirror. Or, if we use a laser collimator, then the spot from the laser should return to where it started from.

Vignetting estimation and a few words about mark on the secondary mirror.

Let's first try to estimate how fast the illumination drops for the case of a zero non-vignetted field. Then we will increase the size of the secondary mirror and see what changes come from this. In this case, we will choose the value of the FFL once at the very beginning and will not change it during the calculations. As was shown earlier, it is possible to use the approximation with a high degree of accuracy ($< 1.5\%$ for aperture less than $f/3$) that our secondary mirror is an ellipse with an aspect ratio of $1{:}\sqrt{2}$. Vignetting in this case will occur when the cone of light goes from the main mirror not directly to the focus point, but at some angle. It is most convenient to illustrate this by looking at our telescope as if from above: so that the ocular unit looks straight down, and we, imagining that our tube is transparent, would see the back side of the diagonal mirror in the form of a circle:

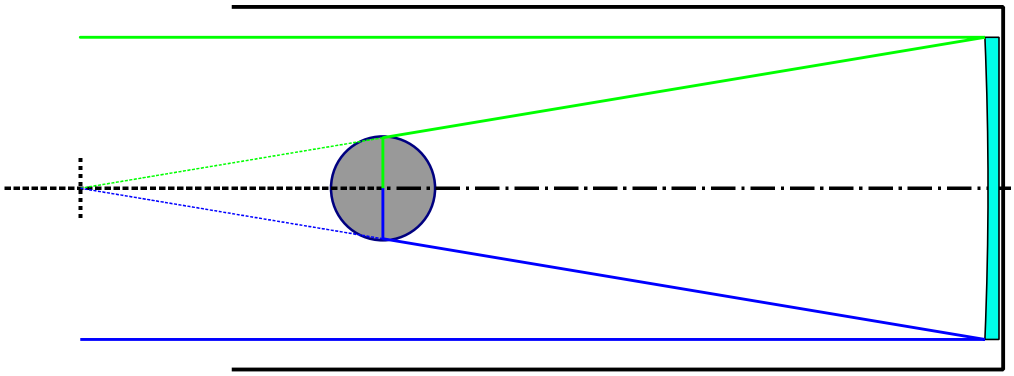

Figure 16 Illustration of the path of the rays when viewed from above.

The problem of vignetting then can be reduced to the problem of the area of intersection of two circles, one of which is formed by the main mirror, and the second is the actual working area of the secondary.

The scheme for this case is shown in Figure 17. To obtain an estimate of vignetting, we are interested in the area shaded in green. In order to find the coordinates of the intersection point, it is easiest to use algebraic equations:

$$\begin{cases} x^2 + y^2 = d_s^2/4 \\ (x-p)^2 + y^2 = d_l^2/4 \end{cases} \qquad 40$$

Where d_s is the transverse diameter of the secondary mirror, $d_l = iD/F$ is the diameter of the light beam formed by the main mirror at a distance corresponding to the length where we break/reflect our main optical axis i, and p is the distance between the centers of the circles.

The intersection point, relative to the first circle, will have a horizontal coordinate:

$$x = \frac{d_s^2 - d_l^2 + 4p^2}{8p} \qquad 41$$

Just do not forget that the value found in this way will only make sense when the distance is either less than half the sum of the diameters, or more than half their difference. Since for the first case the circles are so far away that they do not intersect, and for the second — one lies completely inside the other. By subtracting one equation from another, we simply ruled out the possibility of imaginary roots, but they are there. Well, since we need real solutions, we need to work in an area where: $p \in \left(\frac{d_s - d_l}{2}, \frac{d_s + d_l}{2}\right)$.

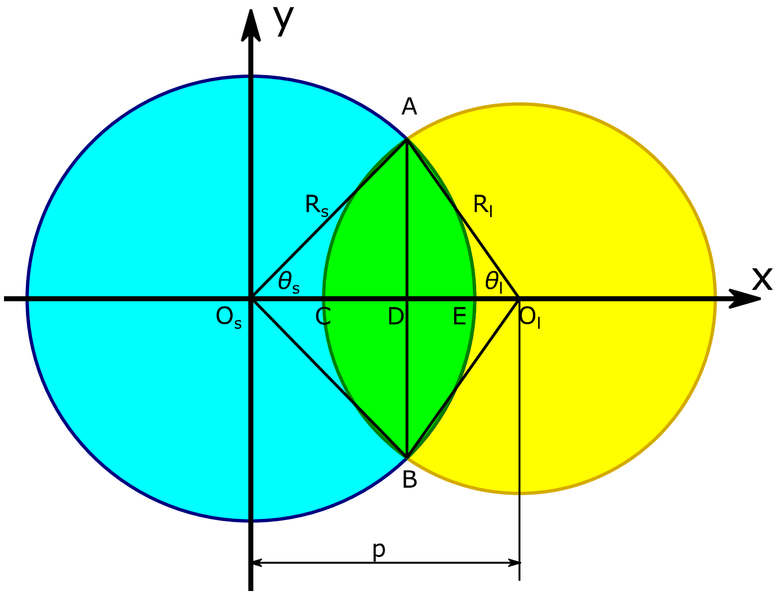

Figure 17 Diagram for two circles intersection.

For angle θ_s we have:

$$\theta_s = acos\frac{x}{d_s/_2} = acos\frac{d_s^2 - d_l^2 + 4p^2}{4pd_s} \qquad 42$$

To find the area bounded by the circumference of the secondary mirror above the oX axis and the vertical line AD, you need to find the difference between the areas of the corresponding sector O_sAE and the triangle O_sAD:

$$S_1 = \frac{\pi d_s^2}{4} \cdot \frac{\theta_s}{2\pi} - \frac{1}{2} x \frac{d_s}{2} \sin\theta_s = \frac{\theta_s d_s^2}{8} - \frac{d_s}{4} \cdot \frac{d_s^2 - d_l^2 + 4p^2}{8p} \sin\theta_s \qquad 43$$

Using Pythagorean trigonometric identity and the fact that cos and acos are inverse of one another:

$$S_1 = \frac{\theta_s d_s^2}{8} - \frac{d_s}{4} \cdot \frac{d_s^2 - d_l^2 + 4p^2}{8p} \sqrt{1 - \left(\frac{d_s^2 - d_l^2 + 4p^2}{4pd}\right)^2} = \\ = \frac{\theta_s d_s^2}{8} - \frac{d_s^2 - d_l^2 + 4p^2}{128p^2} \sqrt{16p^2 d_s^2 - (d_s^2 - d_l^2 + 4p^2)^2} \qquad 44$$

For the second circle with the offset, it is quite similar:

$$\begin{cases} (x+p)^2 + y^2 = d_s^2/4 \\ x^2 + y^2 = d_l^2/4 \end{cases} \\ x = \frac{d_s^2 - d_l^2 - 4p^2}{8p} \qquad 45$$

However, it should be remembered that here, since the intersection point lies to the left of its center — we must take the value of x with the opposite sign!

For the angle θ_l we have the expression:

$$\theta_l = acos\frac{-x}{d_l/_2} = acos - \frac{d_s^2 - d_l^2 - 4p^2}{4pd_l} = \pi - acos\frac{d_s^2 - d_l^2 - 4p^2}{4pd_l} \qquad 46$$

By analogy with the first circle for S_2 we have:

$$S_2 = \frac{\theta_l d_l^2}{8} + \frac{d_s^2 - d_l^2 - 4p^2}{128p^2} \sqrt{16p^2 d_l^2 - (d_s^2 - d_l^2 - 4p^2)^2} \qquad 47$$

The plus sign in the second term appears precisely because our x is negative, and in order to "subtract" the area of the triangle, we must "add" it.

In order to estimate the vignetting itself, we must divide the resulting sum, which is actually half of the desired intersection area, by half of the area of the light beam:

$$\frac{1}{2}\pi\frac{d_l^2}{4}=\frac{\pi d_l^2}{8} \quad 48$$

The last unknown quantity is the offset p for the tilted beam. You can get it in the following way. When moving from the focus point to another point, still in the focal plane and at a distance u, when viewed from the center of the main mirror, the angle of this movement will be equal to:

$$\xi = \operatorname{atan}\frac{u}{F} \quad 49$$

Consequently, the displacement of the center of the circle will be seen at the same angle, but the corresponding distance will be proportionally less than u by:

$$p = u\frac{(F-i)}{F} \quad 50$$

Now we have all the variables to see what the vignetting looks for different optical schemes. For the illustration, I chose two cases with the same aperture of 200 mm and focal ratios of $f/8$ and $f/4$, which will collect light on the matrix 16 times faster than the first. The FFL in both schemes was 200 mm, which may be excessive but reasonable, since 100 mm will be required to bring the beam out of the main mirror, five centimeters for the path of the rays through the tube, the focuser, possibly a filter wheel, and another five — the path of the rays inside the digital camera, be it a SLR or a specialized astro-camera.

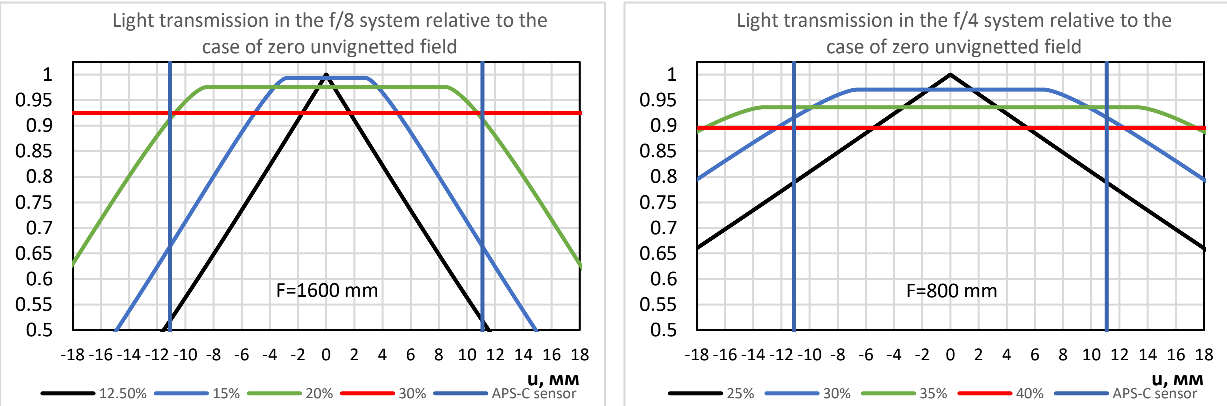

Figure 18 Vignetting for different focal ratios and different sizes of secondary mirrors. Their corresponding linear central obstruction values are shown as a percentage in the plot legend.

It can be seen that fast Newtons have a loss in the value of the central obstruction — the minimum size of the secondary mirror, which uses the entire aperture is greater for them. This leads to more pronounced diffraction effects. For a long-focus Newton, with a very modest central obstruction of 15–20%, the entire area of the main mirror is already effectively used. While on a fast Newton this value starts only from 25%. However, one should not think that the image loses much in brightness, since only a small fraction of the area is screened: at $R = 15 \div 40\%$, only $R^2 \approx 2 \div 16\%$ of light is lost.

The degree of increase in diffraction effects due to central obstruction can be estimated in Figure 19:

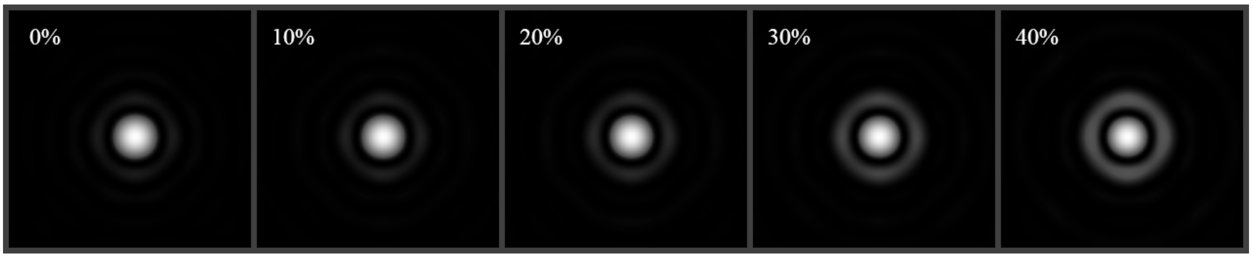

Figure 19 Presence of the first diffraction ring for different central obstruction values.

As a result, for "long and dark" Newtons we have a gain in the purity of the diffraction image, and on the other hand, for "fast and aperture-rich" almost order of magnitude faster signal accumulation.

And now let's estimate which part of the secondary mirror in the center is really inoperative due to the central obstruction. Due to the fact that the beam reflected from the main mirror converges to the focal point, the shadow formed by the secondary has time to decrease in proportion to the value of i/F. That is, the diameter of the shadow is:

$$d_{sh} = \frac{id_s}{F} \qquad 51$$

Expressing the size of the shadow using the telescope aperture k and the central obstruction R, we will have:

$$d_{sh} = \frac{id_s}{F} = \frac{iDR}{F} = ikR \qquad 52$$

Substituting in formula 52 the value of the FFL $200 \div 250\ mm$ and the central obstruction $20 \div 30\%$, we get the diameter of the shadow on the secondary $5 \div 9\ mm$ for aperture $f/8$, and $10 \div 19\ mm$ for $f/4$. And if we remember that this is the diameter of the shadow, and the radius is half as much, then on the camera's matrix this will correspond to the distances from the center of the frame:

$$u = \frac{pF}{2(F-i)} = \frac{pF}{2F\left(1-\frac{i}{F}\right)} = \frac{p}{2(1-c_i)} \qquad 53$$

Where c_i — FFL expressed as a fraction of the focal length. If the FFL is approximately $1/5$ of the focal length, then this will correspond to the coordinates on the matrix $3 \div 6\ mm$ for aperture $f/8$ and $6 \div 12\ mm$ for $f/4$. You can estimate the position of these points, whether they fall into the nonvignetted field, by looking at the graphs in Figure 18. From which we can conclude that if we want to put a central mark on the secondary mirror, we need to make it as small as possible, because almost all off-axis beams will be screened by it. This is especially critical for "dark" Newtons with aperture ratio less than $f/6$. And, moreover, each area of the secondary mirror that does not reflect light is equal to the obstruction of the primary mirror F/i times more! That is, if our mark is a circle, say 5 mm, and if $c_i = 1/5$, this will be equivalent to shading the main mirror with a circle of 25 mm. Which, at the same time, will not coincide with the already existing shadow from the secondary itself, but will be added to it. After I have described how the installation of the mark spoils our lives, we need to quantify it". Do not forget that this will be an addition of the form $\pi d_{mark}^2/4$. If we express it in terms of the non-working area of the primary mirror, then it will be:

$$S_{eff} = \pi\frac{D^2}{4} - \pi\frac{d_s^2}{4} - \pi\frac{d_{mark}^2}{4c_i} \qquad 54$$

It is more convenient, as always, to express this in relative units, where we take as a basis for comparison that we do not have any mark:

$$\frac{S_{mark}}{S_{eff}} = \frac{\pi\frac{D^2}{4} - \pi\frac{d_s^2}{4} - \pi\frac{d_{mark}^2}{4c_i}}{\pi\frac{D^2}{4} - \pi\frac{d_s^2}{4}} = 1 - \frac{d_{mark}^2}{c_i(D^2 - d_s^2)} \qquad 55$$

Substituting here the values for my telescope, for example $c_i = 0.23$, $D = 150\ mm$, $d_s = 44\ mm$ and $d_{mark} = 5\ mm$, I will get vignetting at the level of half a percent for off-axis beams. That is, a completely imperceptible value. Therefore, although the conclusion is that the smaller the mark on the secondary, the better: an additional 5 mm will not play a big role. But why did I take such a large size of five millimeters? The fact is that if we want to use a laser collimator for alignment, we need the working area of the diagonal mirror to remain in the center of the mark. Otherwise, the beam simply will not be reflected further — to the primary one. Therefore, we need the same "sight", as is usually glued to the center of the main mirror: a circle with a hole in the middle. Moreover, since this second mark is much closer to the focus point, it is better to blacken it so as not to create additional illumination if there are any bright objects in the field of view of the telescope. So, our mark will not interfere with observations at night, and during the day, in the light, it can be easily seen as a black outline on the mirror (when looking without eyepiece of course).

Compendium of the formulae which are useful for collimation

Focal ratio of a telescope: the ratio of the diameter of the main mirror to its focal length:

$$k = \frac{D}{F}$$

The minimum size of the minor axis of the diagonal mirror when the aperture of the primary mirror is not yet cut off. Corresponds to zero non-vignetted field $z = 0$.

$$b = \frac{4[D - k(F - i)]}{(4 - k^2)}$$

Offset of the secondary mirror relative to its geometric center in fractions of its major axis:

$$r = \frac{\tan \varphi'}{2} = \frac{D - z}{4F}$$

The value of the same offset in millimeters:

$$d = r(A + a) = \frac{\sqrt{2}[iD + z(F - i)](D - z)}{4F^2 - (D - z)^2}$$

The size of the unvignetted field of view as a function of the transverse size (minor axis) of the secondary mirror b:

$$z = D - \frac{2F\left[(F - i) - \sqrt{[F - i]^2 - b(D - b)}\right]}{b}$$

Tolerance for the focal position:

$$\Delta(\mu m) \lesssim \left(\frac{F}{D}\right)^2 = \left(\frac{1}{k}\right)^2$$

The total required accuracy of alignment of the focuser unit and the diagonal mirror, depending on the size of the desired linear field l:

$$|\beta_{degree}| \lesssim \frac{0.114}{l(mm)}\left(\frac{1}{k}\right)^2$$

List of useful sourses:

http://astro-talks.ru/forum/viewtopic.php?f=8&t=302#p1460
https://www.telescope-optics.net/defocus1.htm
http://prozarium.ru/TextDetails.aspx?TextID=2112